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GEOMETRIC MODELING OF SPATIAL COVERINGS OF BUILDINGS CONSTRUCTIONS BY CHAINS OF SUCCESSIVE SUPERPOSITIONS

In this article we have studied a drafting of a chain of successive superpositions of some pairs of points for modeling of a curve as a part of the frame of a curvilinear surface. It was shown that a superposition of n points of the discrete analog of a given curve can be replaced by a chain of successive superpositions. Also we have got formulas for calculating superposition coefficients of the chain, which allow determining coordinates of unknown nodal points of the curve without making any system of equations.

Keywords: *discrete geometric modeling, covering surface, static-geometric method, geometric apparatus of superpositions, value of recurrent dependence, coefficients of superposition.*

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ГЕОМЕТРИЧНЕ МОДЕЛЮВАННЯ ПРОСТОРОВИХ ПОКРИТТІВ БУДІВЕЛЬНИХ СПОРУД ЛАНЦЮГАМИ ПОСЛІДОВНИХ СУПЕРПОЗИЦІЙ

Проведено дослідження організації ланцюга послідовних суперпозицій пар точок для моделювання кривої лінії як складової каркаса кривої поверхні. Показано, що суперпозиція n точок дискретного аналога заданої кривої може бути замінена ланцюгом послідовних суперпозицій із урахуванням величини рекурентної залежності. Виведено формули обчислення величин коефіцієнтів суперпозиції ланцюга, що дозволяють визначати координати шуканих вузлових точок кривої без складання систем рівнянь.

Ключові слова: *дискретне геометричне моделювання, поверхні покриттів, статико-геометричний метод, геометричний апарат суперпозицій, величина рекурентної залежності, коефіцієнти суперпозиції.*

Introduction. The present-day civil engineering more and more frequently applies covering in the form of complex geometric configuration coats permitting to design unique architectural structures due to the vast variety of shapes. The covering coats' advantages include, firstly, their ability to cover large spans without the intermediate supports, and secondly, ability to cover various plan shaped structures. Calculation of these structures with account of the materials' physical properties, manufacture technology and installation features require information on the object presented in the discrete form. Therefore, it seems reasonable to perform their formation process in the discrete form since the very beginning.

Nowadays, discrete geometrical modeling is the most promising field of the applied geometry development, which can be roughly divided into studies of continuous geometric images discretization and structural morphology (shaping) due to discrete output data.

Surfaces of the coats, having no significant flexion moments, are formed under the impact of their own weight (by gravity). Such surfaces can not be described by an equation. Dead load, being uniformly distributed across the coat's surface, strictly restricts the amount of the surface shape modeling parameters, thus reducing the designer's creative abilities. Therefore, it would be expedient to design curvilinear balanced surfaces using parts of simple surfaces, if they can be joined smoothly.

Staticogeometric method of the discrete geometrical modeling of curved lines and surfaces [3] permits to obtain discrete curvilinear surfaces' frames under the effect of the external formative load and, additionally, is a simple enough and descriptive tool. Using the staticogeometric method permits to obtain discrete curvilinear surfaces' frames with the arbitrary support contour.

A curved line model is easier to study than a surface model. The line model can be transferred to the surface model, which is formed according to the same regulations, if the said line is considered as part of the surface's frame. The discrete surface model's properties can be obtained as a result of generalizing the relevant line model's properties.

Analysis of the latest studies and publication sources. The staticogeometric method's mathematical apparatus is based on solving intricate systems of linear equations, thus complicating the process of computer calculations. The present study is dedicated to the problems of expanding the shape-generating possibilities of the staticogeometric method by means of the mathematical numerical sequences apparatus, permitting, for instance, to avoid making systems of linear equations at discrete images formation [4].

Publications [5, 6] present approaches to determining the discrete analogs of certain functionalities on the basis of the geometrical apparatus of one-dimensional points set superpositions, which also permits formation of discrete images without making and solving intricate systems of equations. Form management of the discretely presented curves (DPC) is performed by means of varying the superposition indexes' values.

A dissertation thesis [7] was devoted to studying the discrete point sets' superpositions, where the possibility of stretched grids' shape management on the functional addition basis was under consideration. Articles [8 – 10] a number of the above superpositions' properties was proved and the conclusion was made about the prospective efficiency of the geometric superpositions apparatus comprehensive astudy. The study [11] demonstrates that superposition of n points can be replaced by a chain of successive superpositions.

Highlighting parts of the general problem that haven't been solved before. The classic method of finite differences, staticogeometric method and mathematical apparatus of numerical sequences have their own advantages and disadvantages in terms of solving specific practical problems. Therefore, their research, enrichment with new efficient algorithms, studying the possibility of their compiling and expanding on this basis the output data sets are topical.

Further development and improvement of the above methods in general are also topical. Using the geometric superpositions apparatus in combination with the said methods permits significant raising the efficiency and expanding the possibilities of the continuous geometric images discrete modeling process.

Studying the methodology of discrete geometrical modeling on the basis of geometric superpositions apparatus, namely, studying possible variants of superpositions chains for the purpose of deducing the analytical correlations of the determined coordinates of the arbitrary curvilinear surface knot as a set knots' coordinates superposition, will permit modeling discrete frames of the curvilinear coatings' balanced surfaces on the complex configuration support contour as well as shape management in the interactive mode without making intricate equations systems and, thus, significant resource saving.

Problem statement. The paper is aimed at studying one of the variants of arranging the point pairs successive superpositions chain for the curved lines discrete modeling, since the curved surface discrete model's properties can just as well be obtained through generalizing of the respected curved line's properties.

Since one of the main principles concerning the staticogeometric fit of curve method is a curved line's shape management by means of changing the external shaping load distribution type [12], it seems reasonable to take into account the load value at forming discrete curves by means of the successive superpositions chains.

At the discrete images formation on the basis of geometric superpositions apparatus, instead of the term "external loading value", it would be more expedient to use the notion of "recurrent dependence value", which will be equal to the external loading value, because the concentrated efforts (the loading value) in the knot points stipulate the balancing efforts in the polygonal line's elements.

Basic material and the results. Formula

$$y_i = k_1 y_{i-1} + k_2 y_{i+1}$$

will be equal to the finite difference three-point dependence [12]

$$2 \cdot y_i = 1 \cdot y_{i-1} + 1 \cdot y_{i+1} ,$$

therefore, the recurrent dependence value, which will be the pre-image of the external shaping loading, for the purpose of forming the discrete analog of the quadric polynomial based on the set knot points superpositions, can be written as follows

$$P_i = y_i - k_1 y_{i-1} - k_2 y_{i+1} ,$$

where P_i is a discrete recurrent dependence value

Provided that $k_1 + k_2 = 1$,

$$P_i = y_i - k_1 y_{i-1} - (1 - k_1) y_{i+1} .$$

Article [11], and article [9] have proved, that superposition of ***n*** points can be replaced with the successive superposition chain.

Superposition *2* of the point pair *1* and *3* of the numerical sequence

$$y_i = 0,2i^2 , \tag{1}$$

which is shown in Figure 1, can be presented (article [8]) as

$$u_2 = u_1 \cdot k_2^{13} + u_3 \cdot \left(1 - k_2^{13}\right) + P_i^2 , \tag{2}$$

where *u* is a generalized symbol of the respected coordinate obtained as a result of the *1* and *3* points superposition;

k_2^{13} is the first of the two indexes of *1* and *3* points superposition for point *2*;

P_i^2 is the recurrent dependence value, which is the analog to the discrete value of the external shaping loading, applied to knot point *2*.

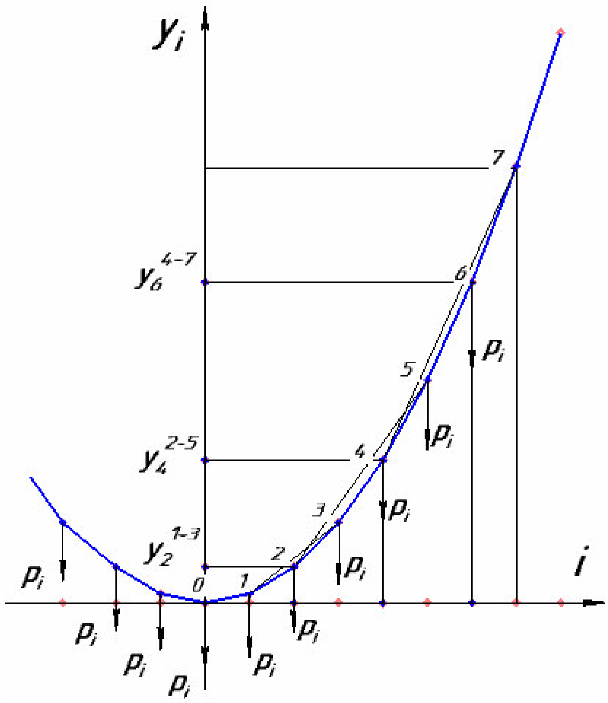

Figure 1 – Discrete analog of the $y_i = 0,2i^2$ numerical sequence as a successive superpositions chain: *1-3*, *2-5*, *4-7*

Let's consider the superposition of the three-points sequence (1)

$$y_4^{1\text{-}3\text{-}5} = k_1 y_1 + k_2 y_3 + \left(1\text{-}k_1\text{-}k_2\right) y_5 \,. \tag{3}$$

Let's show that at a certain dependence between the indexes, superposition (3) can be obtained as a sequence of two superpositions

$$y_2 = k_2^{1\text{-}3} y_1 + \left(1\text{-}k_2^{1\text{-}3}\right) y_3 + P_i^2 \,, \tag{4}$$

$$y_4 = k_4^{2\text{-}5} y_2 + \left(1\text{-}k_4^{2\text{-}5}\right) y_5 + P_i^4 \,. \tag{5}$$

Plugging expression (4) into expression (5), we shall obtain

$$y_4 = k_4^{2\text{-}5}\left\{k_2^{1\text{-}3} y_1 + \left(1\text{-}k_2^{1\text{-}3}\right) y_3 + P_i^2\right\} + \left(1\text{-}k_4^{2\text{-}5}\right) y_5 + P_i^4 \,, \tag{6}$$

and further

$$y_4 = k_4^{2\text{-}5} k_2^{1\text{-}3} y_1 + k_4^{2\text{-}5}\left(1\text{-}k_2^{1\text{-}3}\right) y_3 + \left(1\text{-}k_4^{2\text{-}5}\right) y_5 + \left(k_4^{2\text{-}5} P_i^2 + P_i^4\right) , \tag{7}$$

where $P_4^{2\text{-}5} = k_4^{2\text{-}5} P_i^2 + P_i^4$ is the recurrent dependence value

Comparing expressions (7) and (3), where the respective superposition indexes must be equal, we can write

$$\begin{cases} k_1 = k_4^{2\text{-}5} k_2^{1\text{-}3} \\ k_2 = k_4^{2\text{-}5}\left(1\text{-}k_2^{1\text{-}3}\right) \\ k_3 = 1\text{-}k_1\text{-}k_2 = 1\text{-}k_4^{2\text{-}5} \end{cases} .$$

Therefore

$$\begin{cases} k_1 = k_4^{2\text{-}5} k_2^{1\text{-}3} \\ k_2 = k_4^{2\text{-}5} \text{-} k_4^{2\text{-}5} k_2^{1\text{-}3} \end{cases} ,$$

$$\begin{cases} k_1 + k_2 = k_4^{2\text{-}5} \\ k_1 = \left(k_1 + k_2\right) k_2^{1\text{-}3} \end{cases} ,$$

$$\begin{cases} k_4^{2\text{-}5} = k_1 + k_2 \\ k_2^{1\text{-}3} = \dfrac{k_1}{k_1 + k_2} \end{cases} . \qquad (8)$$

Summarizing the above said, it can be determined that under condition (8) the superpositions pair (4) and (5) is identical to superposition (3).

Superposition of the four points in the sequence (1) is written as

$$y_6^{1\text{-}3\text{-}5\text{-}7} = k_1 y_1 + k_2 y_3 + k_3 y_5 \left(1\text{-}k_1\text{-}k_2\text{-}k_3\right) y_7 . \qquad (9)$$

Therefore:

$$y_2 = k_2^{1\text{-}3} y_1 + \left(1\text{-}k_2^{1\text{-}3}\right) y_3 + P_i^2 , \qquad (10)$$

$$y_4 = k_4^{2\text{-}5} y_2 + \left(1\text{-}k_4^{2\text{-}5}\right) y_5 + P_i^4 , \qquad (11)$$

$$y_6 = k_6^{4\text{-}7} y_4 + \left(1\text{-}k_6^{4\text{-}7}\right) y_7 + P_i^6 . \qquad (12)$$

Plugging expression (10) into expression (11) and (11) into (12), we shall obtain

$$y_4 = k_4^{2\text{-}5} k_2^{1\text{-}3} y_1 + k_4^{2\text{-}5}\left(1\text{-}k_2^{1\text{-}3}\right) y_3 + \left(1\text{-}k_4^{2\text{-}5}\right) y_5 + \left(k_4^{2\text{-}5} P_i^2 P_i^4\right) , \qquad (13)$$

$$y_6 = k_6^{4\text{-}7}\Big\{k_4^{2\text{-}5} k_2^{1\text{-}3} y_1 + k_4^{2\text{-}5}\left(1\text{-}k_2^{1\text{-}3}\right) y_3 +$$

$$+\left(1\text{-}k_4^{2\text{-}5}\right) y_5 + \left(k_4^{2\text{-}5} P_i^2 P_i^4\right)\Big\} + \left(1 - k_6^{4\text{-}7}\right) y_7 + P_i^6 . \qquad (14)$$

On account of formula (14) we can write

$$y_6 = k_2^{1\text{-}3} k_4^{2\text{-}5} k_6^{4\text{-}7} y_1 + k_4^{2\text{-}5}\left(1 - k_2^{1\text{-}3}\right) k_6^{4\text{-}7} y_3 +$$

$$+ k_6^{4\text{-}7}\left(1 - k_4^{2\text{-}5}\right) y_5 + \left(1 - k_6^{4\text{-}7}\right) y_7 + \left[k_6^{4\text{-}7}\left(k_4^{2\text{-}5} P_i^2 + P_i^4\right) + P_i^6 \right] , \qquad (15)$$

where $P_6^{4\text{-}7} = k_6^{4\text{-}7}\left(k_4^{2\text{-}5}P_i^2 + P_i^4\right) + P_i^6$ is the recurrent dependence value.

It follows that

$$\begin{cases} k_1 = k_2^{1\text{-}3} k_4^{2\text{-}5} k_6^{4\text{-}7} \\ k_1 + k_2 = k_4^{2\text{-}5} k_6^{4\text{-}7} \\ k_1 + k_2 + k_3 = k_6^{4\text{-}7} \end{cases} .$$

Therefore

$$\begin{cases} k_2^{1\text{-}3} = \dfrac{k_1}{k_1 + k_2} \\ k_4^{2\text{-}5} = \dfrac{k_1 + k_2}{k_1 + k_2 + k_3} \\ k_6^{4\text{-}7} = k_1 + k_2 + k_3 \end{cases} . \qquad (16)$$

Summarizing the above said, let's consider superposition of ***n*** points of the sequence (1) for this structure of the superpositions chain

$$y_{2n}^{1\text{-}3\text{-}\ldots\text{-}(n\text{-}2)\text{-}n} = k_1 y_1 + k_2 y_3 + k_3 y_5 + \ldots + k_n y_{2n\text{-}1} + \left(1 - k_1 - k_2 - \ldots - k_n\right) y_{2n+1} . \quad (17)$$

Having performed the necessary transformations, we'll obtain:

$$k_{2(1)}^{1\text{-}3} = \frac{k_1}{k_1 + k_2} ,$$

$$k_{4(2)}^{2\text{-}5} = \frac{k_1 + k_2}{k_1 + k_2 + k_3} ,$$

$$k_{6(3)}^{4\text{-}7} = \frac{k_1 + k_2 + k_3}{k_1 + k_2 + k_3 + k_4} ,$$

$$\ldots\ldots\ldots\ldots\ldots\ldots ,$$

$$k_{2n}^{(2n\text{-}2)\text{-}(2n+1)} = \frac{\sum_{i=1}^{n} k_i}{\sum_{i=1}^{n+1} k_i} , \qquad (18)$$

where n is the number of the set chain's point, which coordinates are obtained as a superpositionof the two points coordinates: those of the set point and the one, determined as a result of the appropriately arranged point chain's superposition.

Let's define the formula for calculating the recurrent dependence value in its general form for such arrangement of a superpositions chain.

$$P_4^{2\text{-}5} = k_4^{2\text{-}5} P_i^2 + P_i^4 \ ;$$

$$P_6^{4\text{-}7} = k_6^{4\text{-}7} P_4^{2\text{-}5} + P_i^6 = k_6^{4\text{-}7}\left(k_4^{2\text{-}5} P_i^2 + P_i^4\right) + P_i^6 \ ;$$

$$P_8^{6\text{-}9} = P_6^{4\text{-}7} k_8^{6\text{-}9} + P_i^8 = \left[\left(k_4^{2\text{-}5} P_i^2 + P_i^4\right) k_6^{4\text{-}7} + P_i^6\right] k_8^{6\text{-}9} + P_i^8 \ .$$

Therefore

$$P_4^{2\text{-}5} = k_4^{2\text{-}5} P_i^2 + P_i^4 \ ;$$

$$P_6^{4\text{-}7} = k_6^{4\text{-}7}\left(k_4^{2\text{-}5} P_i^2 + P_i^4\right) + P_i^6 = k_4^{2\text{-}5} k_6^{4\text{-}7} P_i^2 + k_6^{4\text{-}7} P_i^4 + P_i^6 \ ;$$

$$P_8^{6\text{-}9} = k_4^{2\text{-}5} k_6^{4\text{-}7} k_8^{6\text{-}9} P_i^2 + k_6^{4\text{-}7} k_8^{6\text{-}9} P_i^4 + k_8^{6\text{-}9} P_i^6 + P_i^8 \ ;$$

…………………………………………………...…………….;

$$P_{2n}^{(2n\text{-}2)\text{-}(2n+1)} = \prod_{j=2}^{n} k_{2j}^{(2j\text{-}2)\text{-}(2j+1)} P_i^2 + \prod_{j=3}^{n} k_{2j}^{(2j\text{-}2)\text{-}(2j+1)} P_i^4 + \ldots +$$

$$+ k_{2n}^{(2n\text{-}2)\text{-}(2n+1)} P_i^{(2n\text{-}2)} + P_i^{2n} \ .$$

Or:

$$P_{2n}^{(2n\text{-}2)\text{-}(2n+1)} = \sum_{s=2}^{n} \prod_{j=s}^{n} k_{2j}^{(2j\text{-}2)\text{-}(2j+1)} P_i^{2s\text{-}2} + P_i^{2n} \ , \qquad (19)$$

where n is the number of the set superpositions chain's point.

Therefore, for instance, if n =4, according to the formula (19), we'll obtain

$$P_8^{6\text{-}9} = \sum_{s=2}^{4} \prod_{j=s}^{4} k_{2j}^{(2j\text{-}2)\text{-}(2j+1)} P_i^{2s\text{-}2} + P_i^{2n} = \prod_{j=2}^{4} k_{2j}^{(2j\text{-}2)\text{-}(2j+1)} P_i^2 +$$

$$+ \prod_{j=3}^{4} k_{2j}^{(2j\text{-}2)\text{-}(2j+1)} P_i^4 + \prod_{j=4}^{4} k_{2j}^{(2j\text{-}2)\text{-}(2j+1)} P_i^6 + P_i^8 =$$

$$= k_4^{2\text{-}5} k_6^{4\text{-}7} k_8^{6\text{-}9} P_i^2 + k_6^{4\text{-}7} k_8^{6\text{-}9} P_i^4 + k_8^{6\text{-}9} P_i^6 + P_i^8 \ .$$

If $n=5$, then

$$P_{10}^{8\text{-}11} = \sum_{s=2}^{5} \prod_{j=s}^{5} k_{2j}^{(2j\text{-}2)\text{-}(2j+1)} P_i^{2s\text{-}2} + P_i^{2n} = \prod_{j=2}^{5} k_{2j}^{(2j\text{-}2)\text{-}(2j+1)} P_i^2 +$$

$$+ \prod_{j=3}^{5} k_{2j}^{(2j\text{-}2)\text{-}(2j+1)} P_i^4 + \prod_{j=4}^{5} k_{2j}^{(2j\text{-}2)\text{-}(2j+1)} P_i^6 + \prod_{j=5}^{5} k_{2j}^{(2j\text{-}2)\text{-}(2j+1)} P_i^8 + P_i^{10} =$$

$$= k_4^{2\text{-}5} k_6^{4\text{-}7} k_8^{6\text{-}9} k_{10}^{8\text{-}11} P_i^2 + k_6^{4\text{-}7} k_8^{6\text{-}9} k_{10}^{8\text{-}11} P_i^4 + k_8^{6\text{-}9} k_{10}^{8\text{-}11} P_i^6 + k_{10}^{8\text{-}11} P_i^8 + P_i^{10} \ .$$

One can check, if the deduced formulas are correct, having substituted specific numerical values, for instance, of the sequence (1).

Conclusions. The study has demonstrated that superposition of ***n*** points in the discrete analog of the set curve can be replaced with the successive superpositions chain with account of the recurrent dependence value. One of the approaches to determining an arbitrary balanced grid knot coordinates can be studying possible variants of superpositions chains of the set contour knot coordinates and other variants of superpositions chains structures, which will have other dependences between the indexes.

Further studies results will permit modeling discrete frames of the balanced curvilinear coats' surfaces and managing their shapes in the interactive mode without making intricate equations systems.

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